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THE SOLAR RADIATION FIELD AT TWILIGHT

BELOW **100** KM: A SPHERICAL MODEL

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Abstract

Time dependent calculations of minor species distributions require as input the solar radiation field as a function of altitude and solar zenith angle. An isotropic spherical model of the radiation field has been developed to determine the solar radiation field in twilight. Comparison of the spherical model with a plane parallel model of the multiple scattering of radiation at twilight shows that: **(1)** for solar zenith angles less than **95°**, plane parallel solutions are suitable if the initial deposition of solar energy is calculated for a spherical atmosphere; **(2)** for solar zenith angles greater than **95°**, the plane parallel radiation field is several orders of magnitude smaller than that calculated with the spherical model; **(3)** at altitudes above **40** km, the spherical model predicts **10 - 20 %** less radiation than the radiation field calculated with the plane parallel model.



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## I. Introduction

Time dependent calculations of minor species distributions require as input the solar radiation field as a function of altitude  $z$  and solar zenith angle  $\theta_0$ . There has been some concern that results from plane parallel models may be incorrect at twilight. In the present study, an isotropic spherical model of the radiation field has been developed to address the twilight problem. The model does not include refraction effects, ground **albedo**, or aerosols, but does include molecular oxygen ( $O_2$ ) and ozone ( $O_3$ ) absorption, and multiple scattering in an inhomogeneous, spherical atmosphere. Ground **albedo** is relatively unimportant in twilight; refraction effects may be neglected except when making comparison to high resolution measurements; and the presence of aerosols slightly enhance the degree of multiple scattering at all wavelengths. These effects may be included at a later time but the basic results described here will not be affected.

## II. The Models

The model atmosphere employed is the U.S. Standard Atmosphere (1976) with the  $O_3$  profile from Nicolet (1978). The  $O_3$  cross section between 200 and 800 nm is also from Nicolet (1978), as is the  $O_2$  absorption cross section. The model atmosphere is shown in Figure 1 and the  $O_3$  and  $O_2$  cross sections in Figure 2.

The spherical radiative transfer model is patterned after the model developed by Anderson and Hord (1977) for calculation of resonance scattering by atomic hydrogen of the solar Lyman-alpha flux incident on planetary thermospheres and **exospheres**. The basic physical model for **Rayleigh** and **Mie** scattering is described for the plane parallel case by

Anderson and Meier (1979) and Meier et al., (1982). Detailed numerical results for the wavelength region 200-800 nm in the troposphere and stratosphere are described by Nicolet et al., (1982).

Briefly, both the plane parallel and spherical theory used in this study solve the integral equation of radiative transfer for the flux into a volume element  $F$ .  $F$  is normalized to the solar flux at the top of the atmosphere  $F_\infty$ , where  $F_\infty$  is in units photons  $\text{cm}^{-2}\text{s}^{-1}\text{nm}^{-1}$ . The plane parallel model assumes azimuthal symmetry and utilizes Chandrasekhar's (1960) E functions to reduce the problem to one dimension: namely  $F$  as function of vertical optical depth  $\tau$ , or altitude. In the spherical code, azimuthal symmetry is assumed, but  $F$  is now an interlocking function of  $\tau$  and  $\theta_0$ . Thus, on multiple scattering, photons are allowed to migrate in both  $\tau$  and  $\theta_0$ . It will be shown below that the effects of this coupling can be very large in twilight.

### III. Results

#### A. Calculation of $F_0$

When plane parallel models are employed to calculate  $F(\tau, \theta_0)$ , the attenuated direct flux  $F_0(\tau, \theta_0)$  is usually given by

$$F_0(\tau, \theta_0) = \exp[-(\tau + \tau_p) \sec \theta_0] \quad (1)$$

$\tau$  is related in a straightforward way to  $z$  noting that

$$\tau = \sigma_s \int N(z) dz \quad (2)$$

where  $\sigma_s$  is the cross section for Rayleigh scattering, and  $N$  is the neutral atmosphere density at altitude  $z$  in the atmosphere. An absorption which removes a photon from further scattering is designated a pure absorption, and the pure absorption optical depth is

$$\tau_p = \int [\sigma_3 N_3(z) + \sigma_2 N_2(z)] dz \quad (3)$$

where the  $u$ 's are the pure absorption cross sections. If the pure absorber is homogeneously mixed with the scatters, then Chapman functions (Smith and Smith, 1972) may be used in place of  $\sec \theta_0$  in (1) to account for effects on  $F_0$  of **sphericity**. Since  $\theta_0$  is not homogeneously mixed neither Chapman functions, nor  $\sec \theta_0$  is appropriate. Instead, the true slant optical depth must be calculated taking into account the **inhomogeneities** due to ozone. A comparison of (1) with the correct  $F_0$  is shown in Figure 3 at  $\theta_0 = 80^\circ, 86^\circ$ , and  $88^\circ$ , for  $A = 325 \text{ nm}$ ,  $r_p = 0.14$  and  $\lambda = 325 \text{ nm}$ ,  $\tau_p = 0$ , respectively. Clearly, below 50 km, spherical effects are important for  $\theta_0 > 80^\circ$ . No comparisons are made for  $\theta_0 \geq 90^\circ$ , since the plane parallel  $F_0$  is not defined. In Figure 4,  $F_0$  is shown for  $\lambda = 325 \text{ nm}$  and  $\theta_0$  from  $0^\circ$  to  $98^\circ$ . The wavelength  $A = 325 \text{ nm}$  is chosen throughout for illustrative purposes, since both pure absorption and multiple scattering effects are important. However, the model does cover the entire range 200 - 800 nm.

#### B. Calculation of F

Values of F at  $\lambda = 325 \text{ nm}$  are shown in Figure 5 for  $\theta_0 = 60^\circ$  to  $104^\circ$ . The dashed curves are the values of F calculated with the correct spherical  $F_0$ , but the multiple scattering is assumed to take place in a plane parallel atmosphere. It is clear from Figure 5 that for  $\theta_0 < 92^\circ$  the plane parallel solution with a spherical  $F_0$  is adequate to describe the multiple scattering. However, for  $\theta_0 > 96^\circ$  the plane parallel solutions are orders of magnitude lower than the spherical solutions and are off scale.

Another result, not apparent on the log plot in Figure 5, is that above 40 km, F (spherical) falls about 10 to 20% below F (plane parallel) at all  $\theta_0$ . This result has been noted before (Strickland and Anderson,

1973)) and occurs because a photon emitted at a given altitude in a spherical atmosphere has a greater probability of escape from the medium than in a plane parallel atmosphere. As  $z \rightarrow \infty$  the escape probability approaches 0.5 in a plane parallel medium, and 1.0 in a spherical medium.

As an example of the wavelength dependence, Figures 6 and 7 show  $F(\theta_0)$  for  $\lambda = 250$  and  $450$  nm, respectively. At short wavelengths ( $\lambda < 325$  nm) ozone absorbs much of the radiation so that  $F$  is low, and is essentially zero below the ozone maximum. As  $\lambda$  increases beyond 350 nm  $F$  gradually decreases in proportion to the Rayleigh optical depth, and thus the multiple scattering decreases.

#### IV. Discussion

The purpose of this research effort was: (1) to develop a spherical model of the multiple scattering of solar radiation in the twilight atmosphere; and (2) to establish a range of validity for the use of plane parallel models in twilight. The initial model has been developed. A more sophisticated model may be developed in the future which includes such effects as refraction of sunlight, anisotropic scattering by aerosols, ground albedo, and clouds. But the result of comparison with the present model and a plane parallel model suggests that for  $\theta_0 < 95^\circ$ , the plane parallel solution is sufficient, if the proper  $F_0$  is used. This conclusion is stated with the caveat that the spherical solutions are 10-20% lower than the plane parallel solutions at all  $\theta_0$  for  $z > 40$  km.

For  $\theta_0 > 95^\circ$  plane parallel solutions are not satisfactory, and the future improvement of the present model depends to some extent on whether or not the enormous differences in  $F$  between the two models have any effect on the photochemistry in the atmosphere below 100 km.

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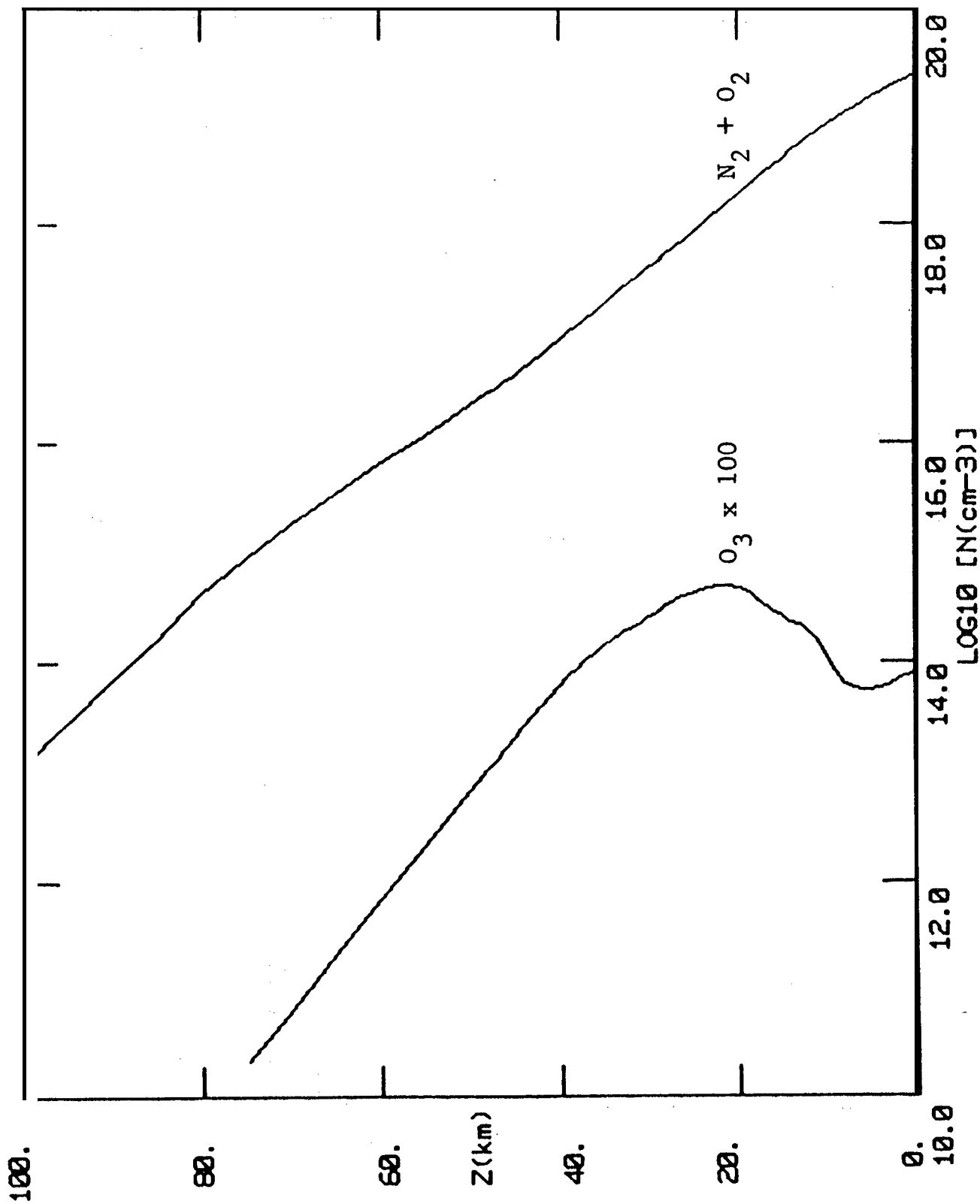


Figure 1. Neutral Atmosphere Density Distribution. Ozone is scaled by a factor of 100.

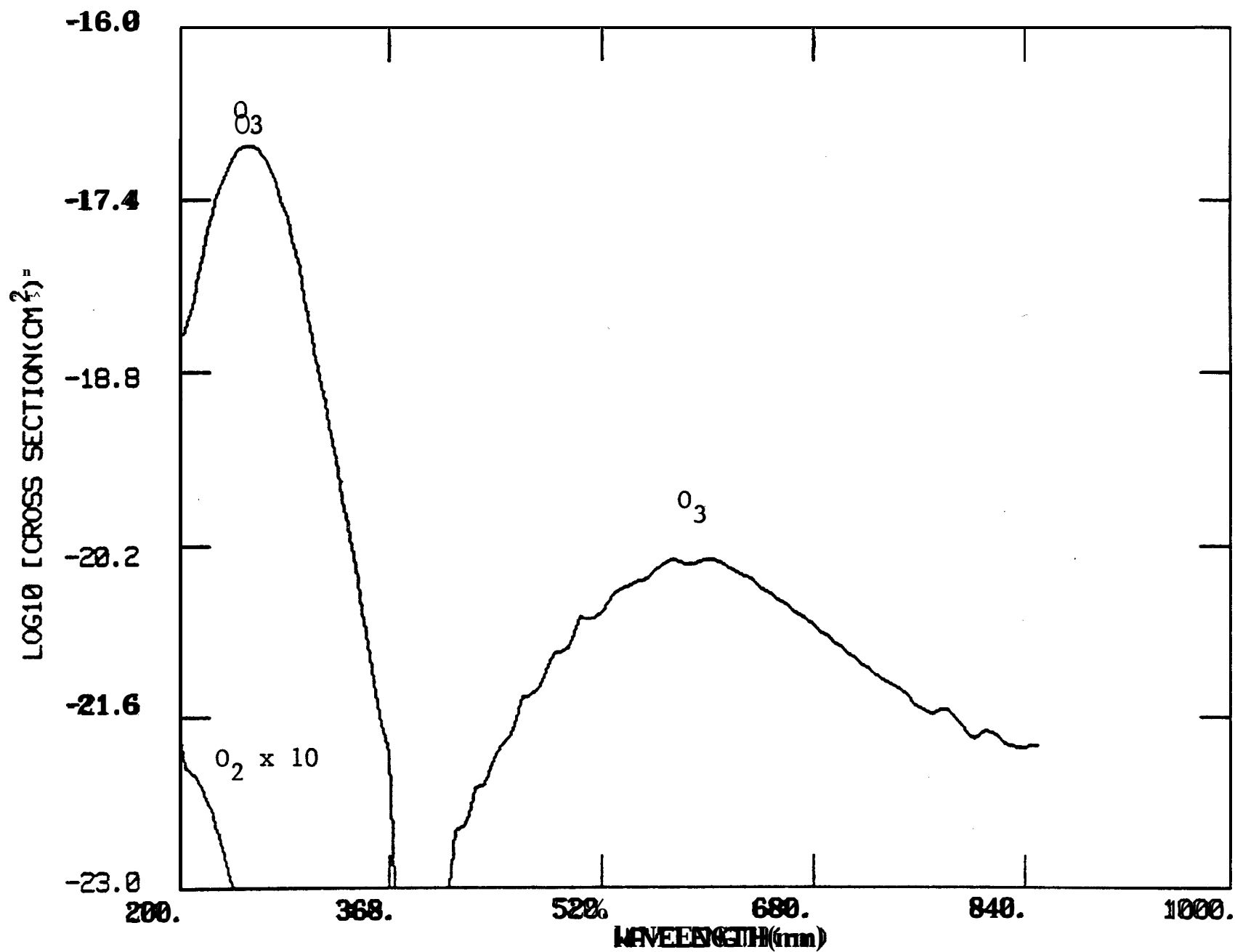


Figure 2.  $O_2$  and  $O_3$  pure absorption cross sections.  $O_2$  cross section is scaled upward by a factor of 10.

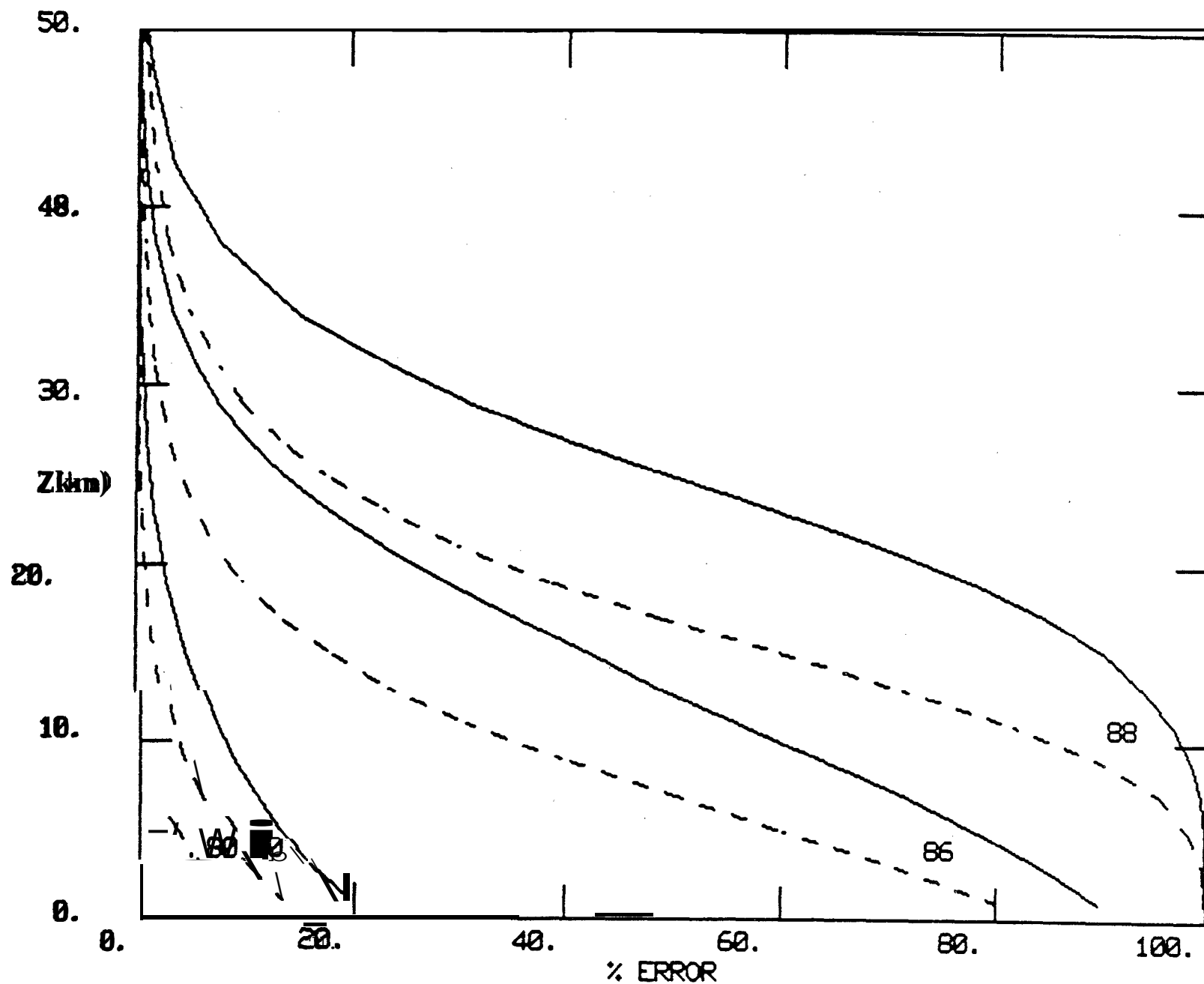


Figure 3. Percent error in plane parallel  $F_0$  when compared to spherical  $F_0$  at  $\lambda = 325$  nm and  $\theta_0$  as indicated. Dashed and solid curves are for  $\tau_p = 0$  and  $0.14$ , respectively.

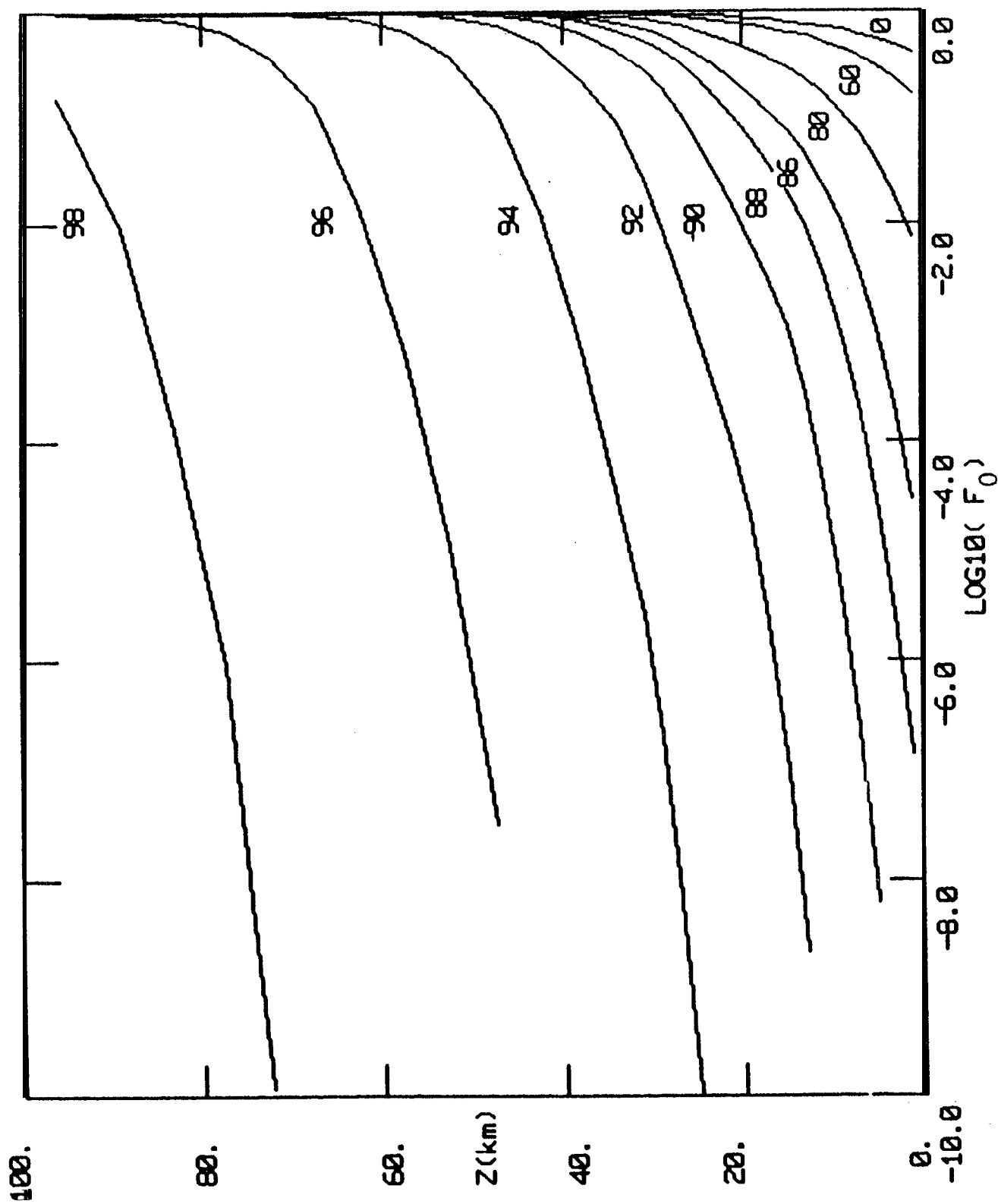


Figure 4. Spherical  $F_0$  at  $\lambda = 325 \text{ nm}$  and,  $\theta_0$  as indicated.

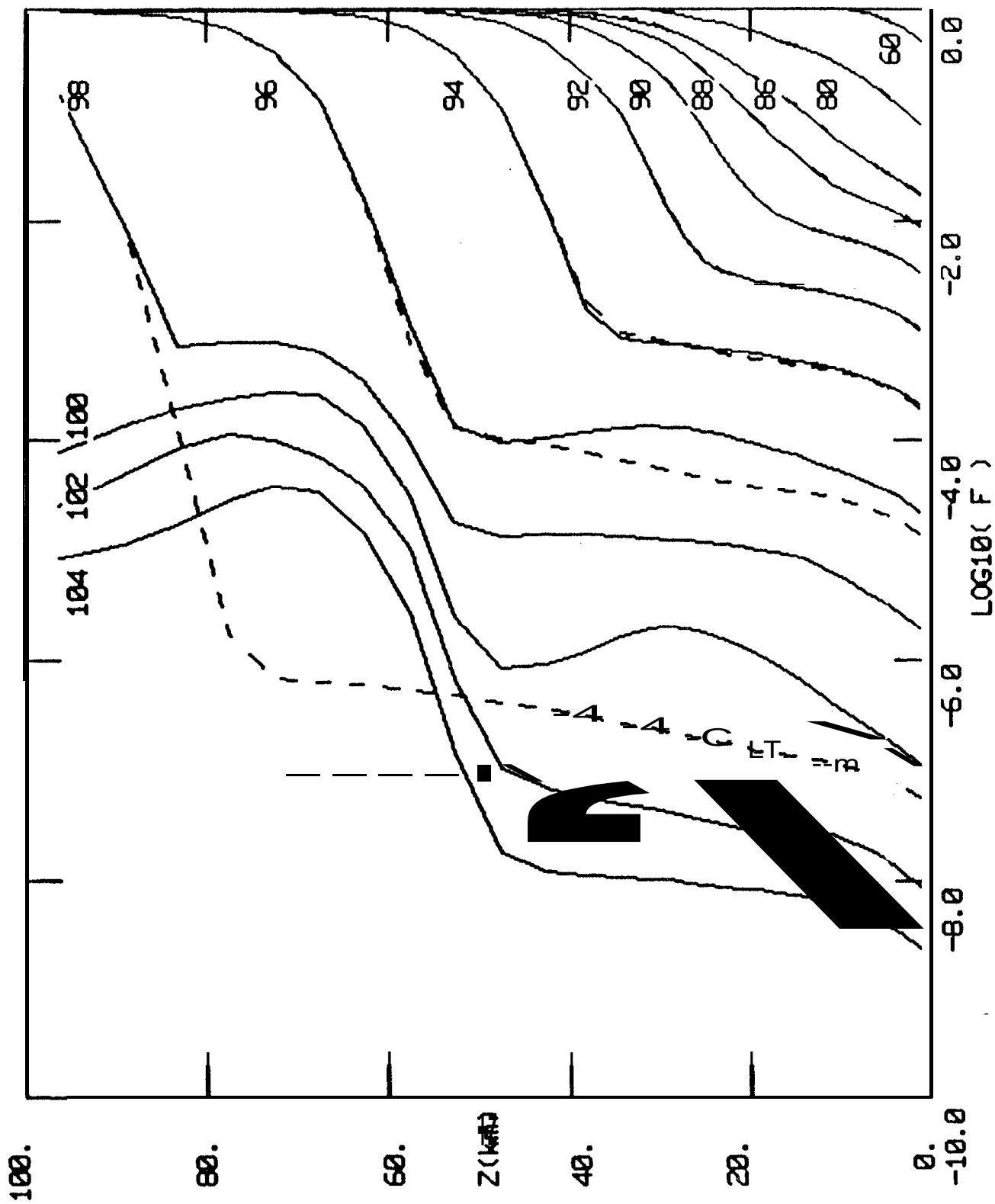


Figure 5.  $F$  as a function of  $\theta_0$  at  $\lambda = 325$  nm. Solid curves are spherical solutions, and dashed curves are plane parallel solutions.  $\theta_0$  as indicated.

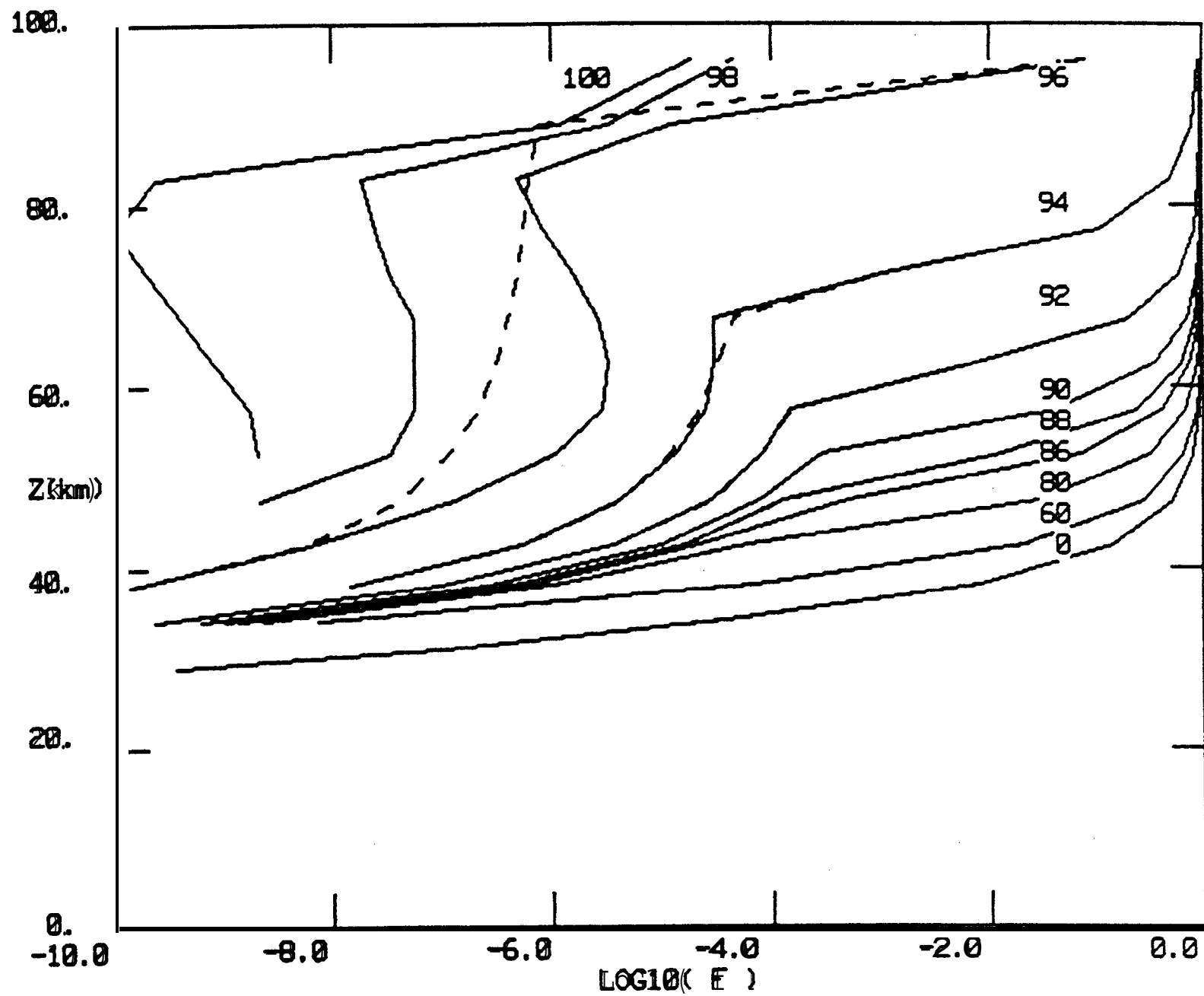


Figure 6.  $F$  as a function of  $\theta_0$  at  $\lambda = 250 \text{ nm}$ . See Figure 5 caption.

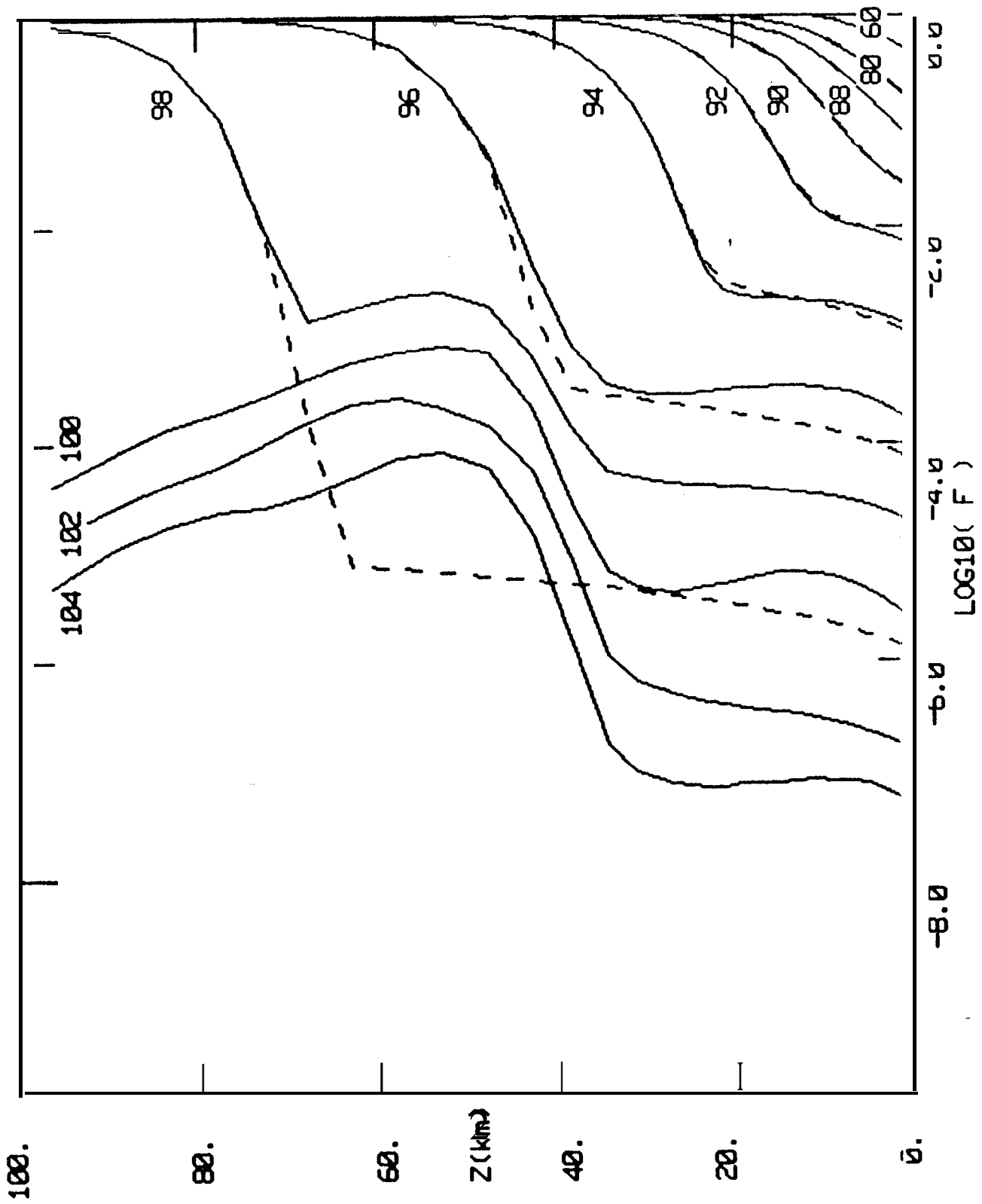


Figure 7.  $F$  as a function of  $\theta_0$  at  $\lambda = 450$  nm. See Figure 5 caption.









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